ESTIMATING SURFACE ROUGHNESS AT SCALES BELOW SENSOR RESOLUTION. Michael K. Shepard, Department of Geography and Earth Science, Bloomsburg University, Bloomsburg, Pennsylvania 17815, mshepard@bloomu.edu.

Descriptive Abstract. We present a simple but physically-based method by which surface roughness (RMS slopes or heights) can be estimated from larger or smaller scales of known topography. The method is based upon the assumption that the surface topography can be reasonably characterized by self-affine or fractal statistics. We present an illustration of the method which estimates the hazards associated with placing a soft lander on Mars.

Introduction. There are many instances in which one may know quantitative roughness characteristics of topography, and from this knowledge, wish to extrapolate the behavior of the topography to greater or lesser scales. One example of this includes determining the relative safety of potential landing sites for planetary missions. Often, one only has topographic information at relatively large scales – tens to hundreds of meters and up. However, landers are most sensitive to topography at scales of a few meters or less. In this brief, we will illustrate a method by which topography at these scales may be reasonably estimated.

Methodology. We assume that topographic parameters are known for a limited range of scales. We also assume that the surface topography is reasonably well characterized by self-affine "fractal" statistics. Over a limited range of scales, it has been well documented that the following properties are observed to apply to most terrestrial and extraterrestrial surfaces (*cf.* [1,2,3,4] for illustrations and a thorough review of the literature):

$$s(x) = s_0 x^{H-1} \qquad (1)$$

$$\xi(L) = \xi_0 L^H \qquad (2)$$

where $s(x) = \tan(\theta)$ is the RMS slope of a surface measured between points spaced a distance, x, apart; s_0 is the RMS slope of that surface at a unit distance spacing; ξ is the RMS height of the surface measured from a profile of length, L; ξ_0 is the RMS height of a profile of unit length, and H is a scaling parameter referred to in this work as the Hurst exponent. For topography, it has been observed that $0 < H < 1$, with H tending to a central value of 0.5. Topography falling into this central category is termed *Brownian.*

If one knows the topography at any scale, x_2 or L_2, and the scaling behavior of that surface, H, one can reasonably estimate the topography at any other scale from Eqs. (1) and (2) [1]:

$$s(x_1) = s(x_2)\left(\frac{x_1}{x_2}\right)^{H-1} \qquad (3)$$

$$\xi(L_1) = \xi(L_2)\left(\frac{L_1}{L_2}\right)^{H} \qquad (4).$$

Below, we give an example and actual application of this result.

Example. Suppose that one wishes to place a lander on Mars. We will assume that the lander has a lateral or base dimension of 3 meters, and an engineering tolerance for landing in terrains with slopes no greater than 10° from the horizontal. Additionally, we assume that the lander has an engineering height clearance of 0.5 meters. Now assume that we are examining two areas as potential landing sites, sites A and B. Site A has good imaging coverage, and a topographic map of the area in interest has been generated with a resolution of 20 m/pixel. Upon analysis, we find that it can be well characterized by a Hurst exponent of 0.5 from scales of several hundred meters down to 20m, and has an RMS slope at 20 m of 2.0°. We apply Eq. (3) and find that the estimated slope of the surface at the 3 m scale is 5.1°. If we make the assumption that the surface is described by Gaussian slope statistics, then we will encounter slopes greater than our engineering tolerance approximately 5% of the time. If we make no assumptions about the distribution of surface slopes, then we must adopt a very conservative estimate (from Chebychev's Inequality [5]) that we will encounter slopes greater 10° no more than 25% of the time.

Site B is also well covered by images from previous missions but the derived topographic products have a resolution of 30m/pixel. However, at this scale the site is found to have an RMS slope of 1.5° and a Hurst exponent of 0.4. Applying Eq. (3), we find that the estimated RMS slope at 3 m scale is 6.0°, somewhat rougher than Site A. Although Site B initially appeared smoother than Site A, this was misleading because of the scale at which it occurred. Additionally, the roughness of Site B increases more rapidly than Site A because of the lower Hurst exponent. Finally, we must always keep in mind that these are extrapolations based upon the observed scaling behavior at larger scales. Our estimate for Site A will be more reliable than for Site B because it is closer to our desired scale (20 m/pixel vice 30 m/pixel).

Caveats. Our experience with topographic data shows that the use of a single Hurst exponent is rarely sufficient to characterize topography over more than two orders of magnitude in scale. More often, there are observed "breaks" in the scaling behavior whereby one value of H is valid for some range of scales, but different values of H are valid at higher and lower scales. As an example, pahoehoe lava flows in Hawaii were observed to have a Hurst exponent of 0.7 for scales from 1m to 10m, but a value of 0.5 for scales less than 1m (site 1 of reference [2]). This break in scaling behavior is attributed to the role of different processes operating on the topography. At the 1-10m scale, topography is controlled by flow rheologic properties – billows and ropy textures are abundant. However, at scales less than 1m, weathering has caused the glassy surface to spall and fragment, littering the surface with innumerable glassy shards. It is therefore of critical importance that the data used for extrapolation be as close as possible to the desired scale. In general, extrapolations from scales more than 1–1.5 orders in magnitude away will be unreliable.

It will also be noted that the engineering height clearances in the above example were ignored. In fact, the RMS height and slope behavior of a fractal surface are functions of one another, and the engineering height and slope constraints are therefore not independent of one another. In general, the RMS slope at some scale, x, and the RMS height from a profile of length, x, are related by [3]

$$\xi(x) = \frac{x\,s(x)}{\sqrt{2}} \qquad (5).$$

In the example above, the lander had a slope tolerance of 10° and a height tolerance of 0.5m. Using Eq. (5), we find that a surface with RMS slope of 10° at a scale of 3m will have an RMS height of 0.37m at the same scale. In other words, the slope tolerance is the more restrictive parameter for site selection in this example.

Application to Mars 2001 Lander. The engineering constraints on the surface roughness for the Mars 2001 lander are: (1) surface tilts of <10° (presumably at the lander scale of ~3 meters) and (2) less than 1% chance of landing on a rock higher than 0.31m (again, presumably for a horizontal scale of ~3 meters). We will assume the surface roughness to be characterized by self-affine fractal behavior and Gaussian height statistics. In this case, the height clearance is the most restrictive engineering constraint. A 1% chance of landing on a 3 meter spot with height of <0.31m is equivalent to requiring an RMS height (1 sigma) of <0.12m for the landing area (0.31÷ 2.58 sigma). Using Eq. (5), we find this corresponds to RMS slopes (1 sigma) of <3.2° at the lander scale, which gives us less than 0.2% probability of landing on a slope >10°. None of this is dependent upon the Hurst exponent.

If we wish to extrapolate this limiting roughness to larger known topographic scales, we must assume some Hurst exponent. Experience has shown most topography to fall between the values of H = 0.3 – 0.7 and so we will adopt these extremes and present a high, low, and intermediate (H = 0.5) scenario. We can use Eq. (3) to estimate the RMS slopes for any horizontal scale and Hurst exponent. Assuming that topographic data is available at the 30m horizontal scale, the maximum tolerable RMS slopes at this scale are 0.64° (H = 0.3), 1.0° (H = 0.5), and 1.6° (H = 0.7). These values would decrease with increasing known horizontal scales (*i.e.,* slopes must be even less at 50m horizontal scales). Additionally, these estimates become less reliable at larger horizontal scales (greater distance to extrapolate).

Conclusions. Natural surfaces have been observed to obey fractal statistics over a wide range of scales. This property provides a way to extrapolate surface properties at scales above and below those which are known and may prove to be of value in estimating lander scale hazards.

References. [1] Shepard et al., *JGR*, 100, 11,709-11,718, 1995. [2] Campbell and Shepard, *JGR*, 101, p. 18,941-18,951, 1996. [3] Shepard and Campbell, Radar scattering from a self-affine fractal surface: Near-nadir regime, *Icarus*, in press, 1999. [4] Helfenstein and Shepard, Submillimeter scale topography of the lunar regolith, *Icarus*, in press, 1999. [5] S. Goldberg, Probability: An Introduction, Dover, 1960.